METHOD AND INSTALLATION FOR COMPLEX DETERMINATION OF THE THERMOPHYSICAL CHARACTERISTICS OF NONMETALLIC SHEET MATERIALS

V. A. Vertogradskii, V. N. Kirillov, and V. A. Efimov

UDC 536.2.083

We present the theory behind the method of complex determination of the thermophysical characteristics, based on solution of the heat-conduction problem for a flat bicalorimeter under conditions of a regular thermal regime of the 2nd kind.

The method of determining the thermal conductivity and heat capacity of nonmetallic sheet materials [1], a development of the work by Eiermann et al. [2] and Yagfarov [3], involves the heating – at a constant rate – of a packet of the sheet materials being investigated, with a plate of known heat capacity inserted between the sheets. Unlike [2], the method proposed by the authors is complex and does not suffer from certain of the difficulties encountered in [3], i.e., the need to ensure an identical flow of heat to the two specimens being heated simultaneously.

Let us examine the analytical solution of the problem.

The temperature distribution in the test specimen and in a control standard (Fig. 1) is found from a solution of the following differential equations:

$$\frac{\partial t_1(x_1, \tau)}{\partial \tau} = a_1 \frac{\partial^2 t_1(x_1, \tau)}{\partial x_1^2}, \quad 0 \leqslant x_1 \leqslant \delta_1; \tag{1}$$

$$\frac{\partial t_2(x_2, \tau)}{\partial \tau} = a_2 \frac{\partial^2 t_2(x_2, \tau)}{\partial x_2^2}, \quad 0 \leqslant x_2 \leqslant \delta_2 \tag{2}$$

for the following boundary and initial conditions:

$$t_1(0, \tau) = t_0 + b\tau; \tag{3}$$

$$\frac{\partial t_2(\delta_2, \tau)}{\partial x_2} = 0; \tag{4}$$

$$t_1(x_1, 0) = t_2(x_2, 0) = t_0 = \text{const} \tag{5}$$

and the joining conditions for the heat flows and temperatures at the point of contact:

$$\lambda_1 \frac{\partial t_1(\delta_1, \tau)}{\partial x_1} = \lambda_2 \frac{\partial t_2(0, \tau)}{\partial x_2}, \tag{6}$$

$$t_1(\delta_1, \tau) = t_2(0, \tau). \tag{7}$$

The solution of the problem is sought in the form of the sum of two solutions

$$t_i(x_i, \tau) = t_{i0} + \tilde{\delta t}_i, \text{where } i = 1, 2. \tag{8}$$

The first solution corresponds to a quasisteady regime; the second solution enables us to account for the effect of the initial temperature distribution.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 16, No. 3, pp. 516-520, March, 1969. Original article submitted April 25, 1968.

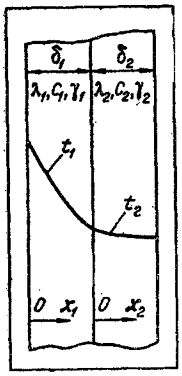

Fig. 1. Temperature field for a two-layer plate.

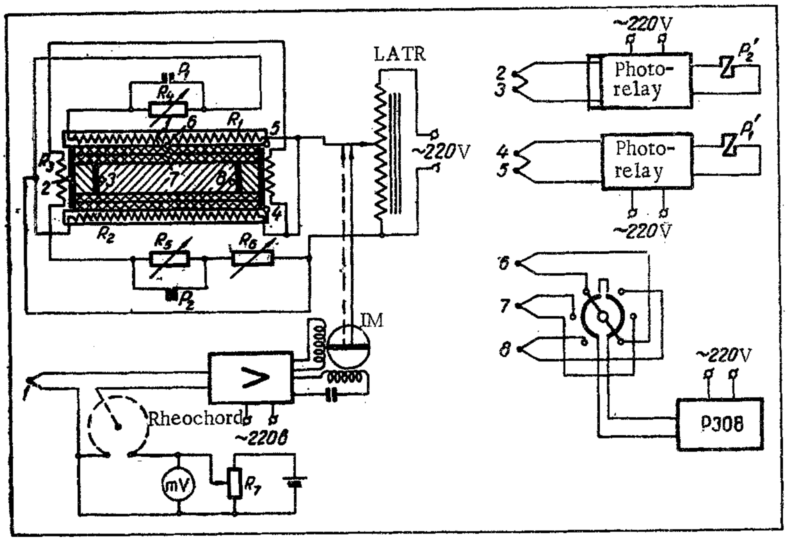

Fig. 2. Electrical circuit of the installation.

The solution corresponding to the quasisteady regime has the form

$$t_{i0} = \frac{b}{2a_i} x_i^2 + c_{1i}x_i + c_{2i} + b\tau. \tag{9}$$

The coefficients c_{1i} and c_{2i} are found from the conditions by means of which we account for the heat balance, and also from Eqs. (3) and (7).

Having used (9) to calculate the temperature differences between the surfaces of the test specimen and its center ($x_1 = \delta_1/2$) and turning to the delay times $\Delta t = b\Delta\tau$, we derived a formula for the determination of the specific heat capacity C and for the coefficient of thermal conductivity λ:

$$C = \frac{4C_2\gamma_2\delta_2(\Delta\tau_1 - 2\Delta\tau_2)}{\gamma_1\delta_1(4\Delta\tau_2 - \Delta\tau_1)}, \tag{10}$$

$$\lambda = \frac{C_2\gamma_2\delta_2\delta_1}{4\Delta\tau_2 - \Delta\tau_1}. \tag{11}$$

Calculating the temperature differences exclusively between the surfaces of the specimen, we find the formula for the determination of λ:

$$\lambda = \varphi C_2\gamma_2\delta_2 \frac{\delta_1}{\Delta\tau}, \tag{12}$$

where

$$\varphi = 1 + \frac{1}{2}\frac{C_1\gamma_1\delta_1}{C_2\gamma_2\delta_2}.$$

A similar formula was derived in [4].

If we do not know the heat capacity of the specimen, we can use this formula if we satisfy the relationship

$$C_1\gamma_1\delta_1 \ll C_2\gamma_2\delta_2,$$

since in this case $\varphi \to 1$.

This condition is achieved through proper selection of specimen and control thicknesses.

The solution by means of which we make provision for the effect of the initial temperature distribution is sought in the form

$$\tilde{\delta}t_1 = \sum_{n=1}^{\infty} A_n \exp(-\mu_n\tau) \sin \sqrt{\frac{\mu_n\delta_1^2}{a_1}} \frac{x_1}{\delta_1}, \tag{13}$$

$$\tilde{\delta}t_2 = \sum_{n=1}^{\infty} B_n \exp(-\mu_n\tau) \cos \sqrt{\frac{\mu_n\delta_2^2}{a_2}} \left(1-\frac{x_2}{\delta_2}\right). \tag{14}$$

These equations satisfy conditions (1), (2), (4), and $t_1(0, \tau) = 0$.

After substitution of (13) and (14) into (8), and after using (6) and (7), we find the characteristic equation for the determination of μ_n:

$$\frac{1}{\varepsilon_1} \operatorname{tg} \sqrt{\frac{\mu_n\delta_1^2}{a_1}} = \frac{1}{\varepsilon_2} \operatorname{ctg} \sqrt{\frac{\mu_n\delta_2^2}{a_2}}. \tag{15}$$

The coefficients A_n and B_n are found in the same way as in [5].

This solution enables us to determine the thermophysical characteristics of the material of the outside layer of a bicomponent body when the volume heat capacity of the material making up the inside layer is known. No limitations are imposed in this case on the relationship between the coefficients of the thermal conductivities of the layers.

In the practical realization of the method in connection with the study of materials exhibiting low thermal conductivity, to eliminate the temperature difference within the control standard it is advisable to use a metal as the standard material. This makes it possible to choose a standard whose heat capacity is known for a wide range of temperatures.

Formulas (13) and (14) enable us to evaluate the time for the onset of the quasisteady regime. If the material of the first plate is a heat insulator, and if the material of the second plate is a metal – and if their thicknesses are commensurate – the time is found from the conditions for the regularization of the temperature field in the insulator (formula (13)). For this case, the time for the onset of the quasisteady regime can be evaluated by the extent to which the following epxression approaches zero:

$$\xi = \frac{\tilde{\delta}t_1|_{x=\delta_1} - \tilde{\delta}t_1|_{x=0}}{t_{10}|_{x=\delta_1} - t_{10}|_{x=0}} = \frac{\tilde{\delta}t_1|_{x=\delta_1}}{\Delta t_1}. \tag{16}$$

This expression has been derived on the basis of (8) and represents the ratio of the two components of the overall temperature difference – the nonsteady and the quasisteady components.

It is obvious that when $0 < \tau < \infty$, $1 > \xi \to 0$.

The numerator in (16) is determined from (13). Series (13) converges rapidly, which follows from the solution of the characteristic equation (15). Limiting ourselves to the first term of series (13), we can present expression (16) in the form

$$\xi = k \exp(-\mu_1\tau), \tag{17}$$

where

$$k \neq f(\tau) < 1,$$

and μ_1 is found from (15).

Having specified the accuracy for the determination of the quasisteady regime at 0.5%, we find $\exp(-\mu_1\tau) \leq 0.005$ or $\mathrm{Fo} \geq 5.3/\varkappa^2$, where $\varkappa = \sqrt{\mu_1\delta_1^2/a_1}$. The calculations which were performed for the heat insulator–metal system, given commensurate thicknesses of the layers ($\delta_1 \approx 2\delta_2$) show that $\varkappa \cong 1$.

To investigate the thermophysical properties of nonmetallic materials by the method described here, we developed an installation which is based on two flat heaters between which a bicalorimeter is positioned; the bicalorimeter is made up of the specimens being tested and of a calorimetric device (Fig. 2).

In connection with the need to measure temperatures when $x_1 = \delta_1/2$, we use two identical flat specimens of thickness d_1. The thickness of the calorimetric device is d_2. Thus, $\delta_1 = 2d_1$ and $\delta_2 = d_2/2$, and the working formulas assume the form

$$C = \frac{C_2\gamma_2 d_2(\Delta\tau_1 - 2\Delta\tau_2)}{\gamma_1 d_1(4\Delta\tau_2 - \Delta\tau_1)}, \qquad (10')$$

$$\lambda = \frac{C_2\gamma_2 d_2 d_1}{4\Delta\tau_2 - \Delta\tau_1}. \qquad (11')$$

The test specimens are disks 90 mm in diameter, with a thickness of d_1 = 1-3 mm.

The calorimetric device is made of M-1 grade copper and it consists of a core surrounded by a protective ring. The core is a disk that is 67 mm in diameter, and its thickness d_2 is 6 mm. The protective ring has an inside diameter of 70 mm, and the outside diameter is 90 mm. The core is held in position by means of 3 thin ceramic tubes.

The selection of copper as the standard material is a consequence of its high thermal conductivity and its thoroughly studied heat capacity.

The side surface of the packet is surrounded by a thin-walled copper screen with a heater. The temperature is regulated automatically to eliminate heat losses through the calorimetric device.

The geometric dimensions of the specimen and of the calorimetric device were chosen on the basis of calculations and from an analysis of the temperature distribution in the system, bearing in mind the exchange of heat between the side surface of the specimen and the screen, proceeding from the condition of permissible distortion of the temperature field in the center portion of the specimen-standard system.

Each heater is made up of a copper rim to which a heating element of Nichrome foil is attached. To ensure reliable thermal contacts, the entire packet is tightened by means of a spring-operated clamp to maintain constant pressure (to reduce the thermal resistance of the contacts, the specimen surfaces are also polished and lubricated).

The linear shape of the heater's temperature curve during the course of the experiment is achieved by means of a programmed device similar to the one described in [6]. The emf of thermocouple 1, positioned in the immediate vicinity of the heating element, is compared with the signal from the device which sets the linearly varying emf – the rheochord. After amplification, the unbalanced signal controls an induction motor (IM) connected to the shaft of an LATR-1 type laboratory autotransformer.

Two-position regulation is accomplished by means of a photorelay based on an M-195/1 galvanometer, a polarized RP-7 relay, and through use of an FD-3 photodiode as a photosensitive pickup. The signal from the differential thermocouple 2-3 is transmitted to the photorelay input; the junctions of the thermocouple are embedded in the screen and in the calorimetric device.

A similar photorelay is also used to maintain heating symmetry for the principal heaters (differential thermocouple 4-5). The screen heater is connected to the output of the autotransformer of the linear-heating system. The regulator resistance is connected in series to the heater to set the power level and the ballast resistance which is bypassed on actuation of the relay. The system is so adjusted that with an open relay 10% of the heater-screen power is released at the ballast resistor.

To measure the temperatures, we used Chromel–Alumel thermocouples. Thermocouple 6 is mounted in the heater rim, and thermocouple 8 is imbedded in the core of the calorimetric device. Thermocouple 7, recording the temperature between the specimens, is rolled to a thickness of 0.05 mm.

The measurements performed during the course of the experiment reduced to the determination of the delay times $\Delta\tau_1$ and $\Delta\tau_2$, in analogy with [7].

The overall error of the method is estimated as approximately 10%.

To test the method on the installation described, we performed experiments with a nonoriented polymethyl methacrylate, and the result from the experiments yielded good agreement with the data of [8].

The installation was used to determine the thermophysical characteristics of numerous materials in a temperature range from 20-40°C. When using low-temperature thermostating devices, we can expand the range of temperature tests to the region of negative temperatures. Moreover, there are no basic limitations on expanding the area of application for the method in the direction of high temperatures.

NOTATION

a	is the coefficient of thermal diffusivity;
t	is the temperature;
τ	is the time;
δ and d	denote thickness;
γ	is the volume mass;
b	is the heating rate;
t_0	is the initial temperature;
$\Delta\tau_1$ and $\Delta\tau_2$	are the temperature delay times for the specimen and the joint with the calorimetric plate relative to the temperature of the heater and to the temperature at the center of the specimen;
ε	is the coefficient of thermal activity;
μ_n	are the roots of the characteristic equation.

Subscripts and Superscripts

1 refers to the parameters of the material being investigated;
2 refers to the parameters of the control-standard material.

LITERATURE CITED

1. V. A. Vertogradskii, V. N. Kirillov, and V. A. Efimov, IV Conference of Young Scientists of the State Scientific Research Institute of the Plastics Industry, Abstracts of Reports [in Russian], Izd. Khimiya, Moscow–Leningrad (1966).
2. K. Eiermann, K.-H. Hellwege, and W. Knappe, Kolloid. Zeitschrift, 174, 2, 134 (1961).
3. M. Sh. Yagfarov, Dokl. Akad. Nauk, 127, 3, 615 (1959); Transactions of the II Conference on Thermography [in Russian], Kazan' (1961), p. 40.
4. Th. Gast, K.-H. Hellwege, and E. Kohlhepp, Kolloid Zeitschrift, 152, 1, 24 (1957).
5. P. Seide, JASS, No. 8 (1958).
6. L. G. Berg, Introduction to Thermography [in Russian], Izd. AN SSSR, Moscow (1961).
7. O. A. Kraev, Tepolenerg., No. 4, 81 (1958).
8. Yu. A. Kirichenko, B. N. Oleynik, and T. Z. Chadovich, Inzh. Fiz. Zh., 5, 70 (1964).